Comment on "Phase Diagram of the Random Energy Model with Higher-Order Ferromagnetic Term and Error Correcting Codes due to Sourlas"

In a recent Letter, Dorlas and Wedagedera (DW) [1] have studied the random energy model (REM) with an additional p-spin ferromagnetic interaction, as a guide to the properties of a p-spin Ising model with both random spin glass and uniform ferromagnetic exchange, itself relevant to an error-correcting code [2]. They showed that the non-glassy ferromagnetic phase, found for $p = 2$ [3] to lie between the paramagnetic and glassy ferromagnetic phases, is squeezed out to larger ferromagnetic exchange as p is increased and is eliminated in the limit of $p \to \infty$. Here we note that (i) we have solved the corresponding problem of a spherical spin system with p-spin glass interactions and r-spin ferromagnetic interactions [4] and have shown that for all $r \geq p > 2$ the opposite situation applies, namely glassy ferromagnetism is suppressed and only non-glassy ferromagnetism remains, and (ii) a simple mapping yields the results of DW and generalizations.

The Hamiltonians for both the Ising and spherical models consist of a disordered and a ferromagnetic term:

$$\mathcal{H} = \sum_{i_1 < i_2 \ldots < i_p} J_{i_1 \ldots i_p} \phi_{i_1} \ldots \phi_{i_p} - \frac{J_0 (r-1)!}{N^{r-1}} \sum_{i_1 < i_2 \ldots < i_r} \phi_{i_1} \ldots \phi_{i_r}\,, \tag{1}$$

where the $J_{i_1 \ldots i_p}$ are independent Gaussian random couplings of zero mean and variance $p! J^2 / 2N^{p-1}$, and $\phi_i^2 = 1$ for Ising or $\frac{1}{N}\sum_i \phi_i^2 = 1$ for spherical spins. The properties of the system can be found from the free energy $f_{\rm SG}(M)$ of the system with $J_0 = 0$ and a constrained magnetization M. They are obtained by minimizing the free energy

$$f(M) = f_{\rm SG}(M) - \frac{1}{r} J_0 M^r, \tag{2}$$

with respect to M, which means solving

$$f'_{\rm SG}(M) \doteq \frac{df_{\rm SG}(M)}{dM} = J_0 M^{r-1}. \tag{3}$$

Generally $f'_{\rm SG}(M)$ is first order in small M, diverges as $|M| \to 1$, and is monotonically increasing in between. For $r = 1$, corresponding to an applied field $h = J_0$, $f'_{\rm SG}(M) = h$, so the equilibrium magnetization increases monotonically with h and tends to unity as $h \to \infty$. For $r = 2$, $f'_{\rm SG}(M) = J_0 M$, so there is always a solution at $M = 0$, and a ferromagnetic solution appears continuously when $J_0 \geq f'_{\rm SG}(0)$. For $r > 2$, the transition is to a magnetization $M_{\rm min} > 0$, and $M_{\rm min}$ increases with r.

The true strength of this method is in predicting the onset of glassiness: this depends on which parts of $f_{\rm SG}(M)$ correspond to glassy solutions and so varies with model and with temperature.

In the upper curve of the Figure we show $f'_{\rm SG}(M)$ for the REM above the glass transition temperature $T_{\rm s}^0$; below $T_{\rm s}^0$ the solution is glassy everywhere. At the temperature shown, the ferromagnetic transition is to a non-glassy phase for small enough r, while for larger r $M_{\rm min}$ is already in the glassy region. As $r \to \infty$, $J_0 M^{r-1}$ approaches a function which jumps from zero to J_0 at $M = 1$, so $M_{\rm min} \to 1$ and the transition is directly to the glassy ferromagnet.

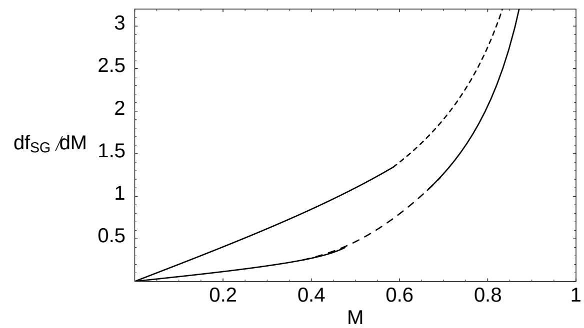

FIG. 1. Plots of $df_{\rm SG}/dM$ versus M for the random energy model with $T/J = 0.7$ (upper curve) and for the spherical p-spin model with $p = 4$ and $T/J = 0.55$ (lower curve). The solid lines represent non-glassy solutions, the dashed glassy.

In the lower curve we show $f'_{\rm SG}(M)$ for the spherical p-spin model slightly above its $T_{\rm s}^0$; at some higher temperature the glassy region disappears; below $T_{\rm s}^0$ the glassy region extends down to $M = 0$. At the temperature shown, for small enough r, $M_{\rm min}$ lies in the lower non-glassy branch, so increasing J_0 leads to a non-glassy ferromagnet, then a glassy, then back to a non-glassy. For some larger r the first non-glassy ferromagnet disappears, and for still larger r so does the glassy ferromagnet. A full calculation [4] shows that the second critical value is $r = p$.

The discussion here has been of the static spinodal transition, but it can be easily extended to the thermodynamic transition by comparing the free energies (2) of competing phases; DW concentrate on this latter case.

Peter Gillin and David Sherrington
Theoretical Physics, 1 Keble Rd, Oxford, OX1 3NP, UK.

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